

Statistical Mechanics of Stock Price Dynamics: Brownian Random Walk and Solution to Fokker-Plank Equation

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ABSTRACT

In this article we discuss some aspects observed in the empirical analysis of stock price dynamics in financial markets. Specifically we consider (i) the behavior of the return probability density function and (ii) the content of economic information in financial time series.

Keywords: Econophysics, Brownian motion, Fokker-Plank equation.

1. INTRODUCTION

At the present time the analysis and modeling of financial markets have become an important research area of economics and financial mathematics¹. The knowledge of the statistical properties of price dynamics in financial markets is necessary for any theoretical modeling aiming to obtain a rational price for a derivative product issued on it¹⁷ and it is the starting point of any valuation of the risk associated with a financial position¹⁸. Moreover, it is needed in any effort aiming to model the system. In spite of this importance, the modeling of such a variable is not yet conclusive. Several models exist which show partial successes and unavoidable limitations. In this paper,

the Fokker – Plank equation is derived and solved for the price returns of a stock based on random walk theory.

This article briefly discusses some of the Brownian Random walk model of Stock price dynamics and Fokker- Plank equation for probability distributions of returns. Starting from these results, one can devise studies trying to enrich and expand this knowledge to provide theoreticians and computer scientists the empirical facts that need to be explained by their models progressively proposed. The ultimate goal is to contribute to the search for the simplest possible model describing a financial market, one of the most intriguing complex systems".

The stock price change dynamics in financial markets is described by an unpredictable time series. Though the stochastic dynamics of stock price time series is yet to be described satisfactorily, the simplest assumption is a random walk with independent identically distributed

increments. Indeed the stochastic process is much more complex than a customary random walk. One key question in the analysis and modeling of a financial market concerns the independence of the price time series of different stocks traded simultaneously in the same market.

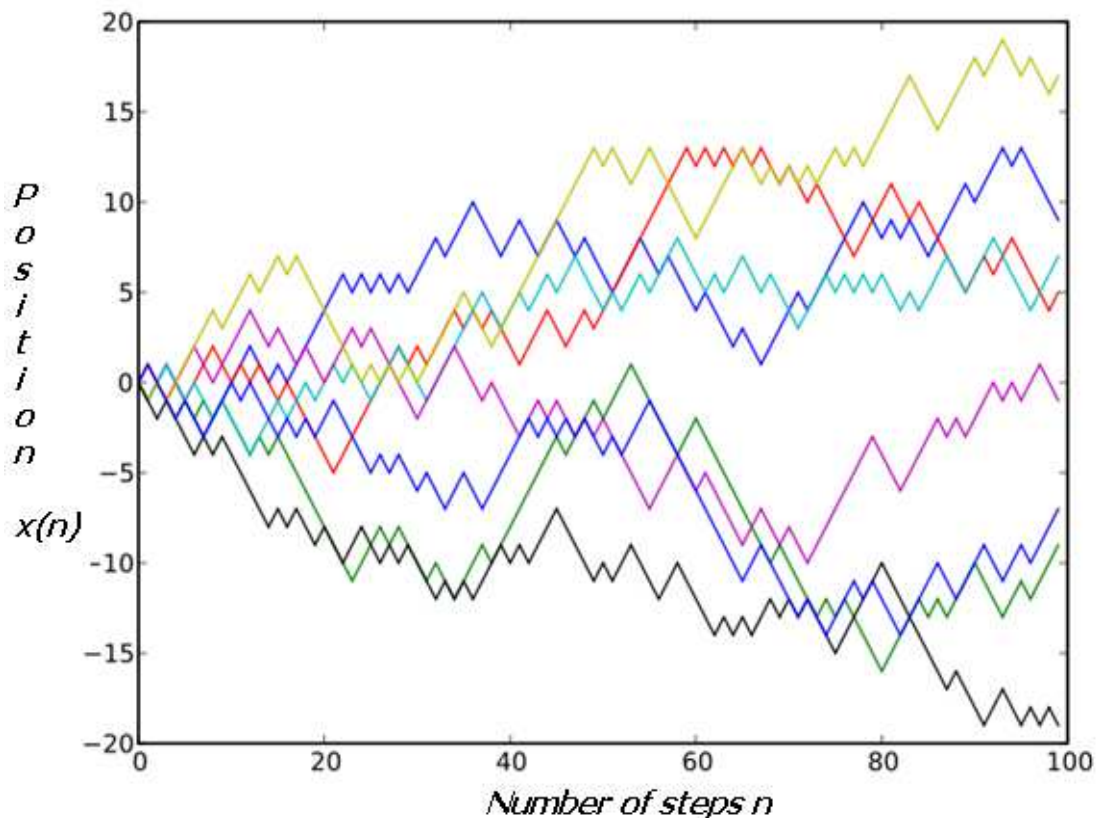


Fig1: Computer simulation for random walk using Fokker plank Equation

FUTURE DIRECTIONS

The actual probability distribution of returns differ significantly from the probability calculated through Fokker Plank equation. Actually the probability of extreme

events calculated through Fokker Plank equation is found to be much smaller than what is observed in stock price returns. The presence of cross-correlations between pairs of stocks has been known since a long time and it is one of the basic assumptions of the

theory for the selection of the most efficient portfolio of stocks³⁹. Recently, physicists have also started to investigate theoretically and empirically the presence of such cross-correlations. It can be a meaningful economic taxonomy that is obtained by starting from the information stored in the time series of stock price only.

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